Morphology and density of ice accreted on cylindrical collectors at low values of impaction parameter. II: Rotating deposits

By F. PRODI1, L. LEVI2, O. B. NASELLO3 and L. LUBART4

Istituto FISBAT-CNR, Gruppo Nubi e Precipitazioni, Via De' Castagnoli 1, 40126 Bologna, Italy

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SUMMARY

The morphology and density of deposits grown on rotating cylinders of 0.5 cm radius, collecting droplets of median volume radius \( \bar{r} = 10 \mu m \), are studied. The wind speed and temperature are regulated so that Macklin's parameter and Stokes number are \( 0.5 < X < 7 \mu m \ m^{-1} \ K^{-1} \) and \( 0.8 < K < 3.2 \) respectively. The knobby structure typical of rotating deposits is shown to form after the formation of the first 0.5-1 mm rime layer. Pronounced lobes, separated by wide entrances are observed at \( K < 3 \). Entrances change into air gaps for larger \( K \) values, as is shown also by comparison with previous results obtained for \( K > 4 \).

Rime density measurements are made by the X-ray micrography technique. The first rime layer and the lobe interior density is represented by a \( p(X) \) curve located between those of Prodi and Levi and Macklin. The former is approached by the experimental point at \( X = 6.5 \), \( K = 3.2 \). The mean deposit density is evaluated by averaging the results obtained by scanning the X-ray micrographs, at different distances \( d \) from the point of deposit initiation. The mean density is shown to decrease markedly with \( d \) for \( K < 3 \), as a consequence of the knobby structure, so that the average density of deposits 3-4 mm thick approaches Macklin's curve. This effect is very much reduced for \( K > 4 \), where lobes are separated by thin air gaps. These results are discussed by taking into account the shadow effect, and its dependence on \( K \). Formulae for \( p(X) \) for different values of \( K \) and deposit thickness are proposed.

1. INTRODUCTION

During the last few years, the study of the density of ice accreted on freely tumbling or rotating bodies has been gaining interest because of the view, put forward by some authors (Pflaum 1984; Prodi et al. 1986a) that the stage in hailstone growth during which the density of accreted ice is substantially below the bulk ice density could play an important role in the stone's development. As a consequence, this density parameter has been taken into account in the formulation of hailstorm cloud models (Farley 1987) and in calculations of hailstone trajectories in a wind field (Heymsfield 1983; Rasmussen and Heymsfield 1987).

However, there are still open questions as regards the ice density behaviour as a function of the growth parameters, especially when it is taken into account that the riming process occurs over the whole collector surface, where these parameters can vary to some extent. Actually, in the first works on this subject (Macklin 1962; Buser and Aufdermaur 1973) the density of ice accreted on rotating cylinders has been represented as a function of Macklin's parameter, \( X = -\bar{r} v_o / T_d \), where \( \bar{r} \) is the median volume droplet radius, \( v_o \) the droplet impact speed in the stagnation region and \( T_d \) the deposit temperature (°C). However, in more recent works, \( v_o \) has been replaced by a mean impact speed \( v_i \), obtained by averaging the local impact speed for each point of the collecting surface, defined as the normal component of the droplet speed. Consequently, different \( p(X) \) equations have been proposed, which are not strictly comparable with each other. On the other hand, the density measurement methods have also been varied in successive studies, some giving the mean density of the deposit considered as a whole.

1 Also: Dipartimento di Fisica, Università di Ferrara, Ferrara, Italy.
2 Permanent affiliation: CONICET, Consejo Nacional de Investigaciones Científicas y Técnicas, Buenos Aires, Argentina.
3 Permanent affiliation: FAMAF, Universidad de Córdoba, and CONICET, Córdoba, Argentina.
4 Permanent affiliation: Servicio Meteorológico Nacional, Buenos Aires, Argentina.
(Macklin 1962, Pflaum and Pruppacher 1979), others the local density in limited areas (Buser and Aufermaur 1973; Prodi et al. 1986a,b).

Given the morphology variations of the deposits and the relations existing between density and morphology, this makes it difficult to represent the results in a well-organized frame, convenient for application in a wide range of different conditions, as can occur in natural phenomena.

Considering the need for improving the understanding of the phenomenon, a basic study of the accretion process was previously performed on fixed deposits (Levi et al. 1991, hereafter referred to as LNP). However, the results obtained in this work cannot be directly applied to the simulation of the growth process of hailstones, mostly showing a spheroidal shape which indicates rotation, and of deposits grown on non-rigid structures, such as electrical cables, which can rotate during growth because of the effect of load distribution.

Taking these considerations into account, a new series of experiments was performed using rotating cylinders as collectors and analysing both the density and the morphology of the deposits. Following LNP, the X-ray micrography method has been applied (Prodi 1970; Prodi et al. 1986a) to evaluate the local density, measured in selected areas, as well as the mean density of the whole deposit.

Considering the importance for deposit morphology of the Stokes number, 

\[ K = \frac{2\rho_w R^2 V (9\eta R)^{-1} }{ } \]

revealed in the study of fixed deposits, the phenomenon was investigated as a function of both \( X \) and \( K \), these parameters varying in the ranges \( 0.5 < X < 6 \mu m \ m s^{-1} \) K and \( 0.8 < K < 3.2 \), respectively. Since in this range of \( K \), lower densities and more open deposit structures were obtained than those reported by Prodi et al., who worked in a range of \( X \) partially overlapping the present one but with \( K > 4 \), the effect of higher values of the latter parameter was also investigated. For this purpose, a few X-ray micrographs of deposits grown by Prodi et al. were re-analysed and the results compared with those of the present experiments.

The analysis of the effects of variations of \( X \) and \( K \) separately on the density values is also interesting in relation to a recent paper (Jones 1990) published when the present one had already been submitted for publication. Actually, in that paper, an empirical formula is given for the rime density as a function of the main parameters that characterize the system. Among these, \( K \) appears explicitly, but the possibility of a special role for this parameter, independent of its effects on the collection efficiency and on the droplet impact speed, which also affect the parameter \( X \), is not considered, and so it remains open to discussion.

2. EXPERIMENTAL METHODS

The experimental arrangement used in the present work was the same as that already described by LNP, with the only difference that the cylindrical collector, placed in the working section of the wind tunnel, was made to rotate at a speed of 4 Hz. The cylinder, of radius \( R = 0.5 \) cm, consisted of a plastic rod coated with ice, with a temperature sensor located below the ice coating and connected to a recorder.

The resulting record showed that the temperature increased rapidly when injection was started and then remained nearly constant during the relatively short growth period, corresponding to a deposit thickness \( d \leq 5 \) mm. Consequently the temperature reading was considered to be the same as the temperature, \( T_d \), of the collecting surface. The experiments were performed in an air-temperature \( (T_a) \) range between \(-22\) and \(-18^\circ C\), while \( T_a \) was less than \(-5^\circ C\). The wind speeds, measured by a hot-wire anemometer, were \( V = 3, 4.5, 7 \) and \( 12 \) m s\(^{-1}\). The droplet spectrum was of the same type as that
described by LNP, the median volume droplet radius being \( r = 10 \mu m \), with fluctuations of about \( \pm 1 \mu m \). The liquid water content, \( W \), varied between 1 and 2 g m\(^{-3} \).

Since the amount of water transported by droplets with \( r \approx \bar{r} \) represented a substantial component of the total water flux, the value of \( \bar{r} \) did not differ significantly between the glass slide, the air and the stagnation region of the cylinder.

The same precautions were taken in the preparation of the deposit sections as in the previous work, the treatment with aniline described by LNP being essential to preserve, in the low-density ice, the structure of lobes which typify the rotating deposits.

The X-ray micrographs of the deposit sections were analysed using the optical densitometers described already, so that the ice density was derived from the densitometer reading, \( \Delta l \), and from the proper calibration (see LNP). In the present case, since the knobby structure of the deposits could determine large local density variations in the same accretion, the densitometer was used, which allowed measurements to be made in selected areas by centring the circular section of the light beam, 1 mm in diameter, on points located near the lobe bases, where it was possible to avoid including, in the zone covered by the beam, air entrances separating lobes. Several readings of \( \Delta l \) were taken in each case and these were compared with the reading of \( \Delta l \), corresponding to the internal bulk ice ring existing in each section (or bulk ice ring fraction when the section was broken). This was the section of the ice coating on the cylinder, on which rime was collected.

For a few micrographs, the densitometer provided with a rotating plate was also used, in order to inspect the density variations occurring in knobby deposits. In this case, the micrographs were scanned by the densitometer’s light beam along circles of increasing radius, concentric with the internal bulk ice ring of the section. For this analysis it was preferred to use a light beam of 70 \( \mu m \) diameter, so as to combine the highest possible resolution with a convenient instrumental sensitivity.

The density, \( \rho \), was measured in g cm\(^{-3} \) and Macklin’s parameter \( X \) in \( \mu m \) m s\(^{-1} \) K\(^{-1} \). These units will be indicated in the diagrams presenting \( \rho \) as a function of \( X \) but will not be repeated in the text.

3. Results

(a) Deposit morphology

A typical characteristic of low-density deposits grown on rotating cylinders is their knobby structure, as shown by photographs and X-ray micrographs by Prodi et al. (1986a). Several aspects of this morphology are analysed in this section because of their relevance to ice-density results, reported herein and also previously by other authors.

The features typifying the lobe structure of a low-density deposit grown at \( K < 1 \) are shown in Fig. 1. The cylinder in Fig. 1(a) has been illuminated so as to highlight both the depth of entrances separating lobes, visible in the lower, partially shaded area of the deposit, and the contrast between this internal structure and the apparent slight roughness, visible in the directly illuminated upper edge of the deposit. Inspection of the lower deposit surface also shows that lobes are completely surrounded by entrances, so that they can be considered to develop about approximately radial axes.

The characteristics of the deposit’s knobby structure, as shown by normal deposit sections, can be seen in the reflected-light photograph of Fig. 1(b) and in the X-ray micrograph of Fig. 1(c), corresponding respectively to the deposit in Fig. 1(a) and to another larger one, grown in similar conditions. These figures indicate that the process is initiated by a rather uniform rime layer consisting of a nearly continuous sequence of
Figure 1. Morphology of deposits grown with $K = 0.8$, $V = 3\, \text{m s}^{-1}$, $T_d < -8^\circ\text{C}$: (a) Photograph of a deposit as grown ($T_d = -12^\circ\text{C}$). Shadows in the lowest part of the deposit show entrances between lobes; (b) reflected-light photograph of a partial normal section of the same deposit; (c) X-ray micrograph of a normal section of a deposit grown with $T_d = -16^\circ\text{C}$. The dark internal ring is the section of the bulk ice coating.
short feathers. When the rime layer reaches a thickness of 0.5–1 mm some of these feathers begin to grow faster, and develop into lobes. These then develop laterally, owing to the growth of ice feathers inclined at about 50 degrees to the lobe axis, thus hindering droplet collection at their sides and determining the periodic lobe distribution.

In Fig. 2, X-ray micrographs are given for two sections of a deposit grown at $K = 1.6$. In this case, the different development of the sections was due to a lack of uniformity in the water injection along the cylinder axis. Figure 2(a) shows that while the density of the deposit is somewhat higher than in the previous example, because it results from the darker grey of the section imprint, its morphology is not substantially different. On the other hand, the comparison of Figs. 2(a) and 2(b) can be instructive regarding the analysis of the process of lobe generation. Actually, Fig. 2(b), typical of a less advanced stage of growth, shows that, at this stage, lobes are just beginning to appear and the deposit consists of a rime layer of nearly uniform thickness. It can be seen that it is very similar to the initial rime layer forming the first millimetre of the more developed section in Fig. 2(a).

![Figure 2](image)

Figure 2. Morphology of a deposit grown with $K = 1.8$, $V = 7 \text{ m s}^{-1}$, $T_d = -13^\circ\text{C}$: (a) X-ray micrograph of a section showing well-developed lobes; (b) X-ray micrograph of a section showing the morphology of a thin rime layer.

Figure 3 shows the morphology of a deposit grown at $K = 3$. In this case the effect of the increase in $K$ is made evident by the narrower spaces between adjacent lobes, the separations between which are marked only by thin air gaps. Despite the higher deposit density, the feathery structure of the lobes is still visible in this micrograph, the angle between feathers and the lobe axis being about 25 degrees.

A comparison of the examples in Figs. 1 to 3 also indicates that the angular frequency of lobes increases along with $K$, the number, $N$, of lobes in a 90-degree arc varying from $N = 4$ for $K < 1$, to $N = 9$ for $K \approx 3$. The latter observation can be considered analogous to that reported in Prodi et al. (1986a) on the relation between $N$ and wind speed $V$, because, when $\bar{r}$ and $R$ are constant, $K$ and $V$ only differ by a numerical factor. In order to compare both groups of results, which were obtained using similar collectors of initial radius $R = 0.5 \text{ cm}$, but different droplet spectra, we present in Fig. 4 the $N(V)$ curve of Prodi et al. (1986a), corresponding to $\bar{r} = 18 \mu\text{m}$, and that which can be drawn on the basis of the present results, corresponding to $\bar{r} = 10 \mu\text{m}$. Note that the latter runs below the former, the difference being about 30%.
Since the experimental conditions in the present work differed from those reported in the previous paper by the lower value of $\bar{r}$, $N$ has also been plotted in Fig. 5 as a function of $K$ and $K/\bar{r}$, namely of $\bar{r}^2V$ and of $\bar{r}V$. It can be seen that both sets of values could be represented by a single curve in the last case. Though more experiments, performed in a wider range of different conditions, would be needed to establish the range of validity of the observed relation between $N$ and $K/\bar{r}$, it is interesting to observe that, as far as Fig. 5(b) is valid, the lobe frequency can be considered to be a reciprocal function of $\bar{r}$, for a given value of $K$. It also follows that the smaller droplets would result in thinner air gaps between lobes, giving place to a more uniform deposit structure.

(b) Deposit density

(i) Density of the first rime layer. As indicated in section 2, local density measurements were taken in the first millimetre of rime, with the densitometer light beam centred on
the lobe axes. The results are plotted in Fig. 6, as a function of the parameter $X$. It can be seen that the experimental points obtained in this way are distributed in the space between Macklin’s curve (a):

$$\rho = 0.11X^{0.76}$$  \hspace{1cm} (1)

representing the mean density of rotating cylinders, and Prodi and Levi’s curve (b):

$$\rho = 0.28X^{0.6}$$  \hspace{1cm} (2)

which was obtained in the range $3 < X < 7$ for the local density in the first rime layer of both fixed and rotating deposits. This curve has been extrapolated here so as to include the low-density range studied in the present work.

For a more exhaustive comparison of the present results with previous ones, we have also considered here those reported by Pflaum and Pruppacher (1979) who measured, in the range $X < 2$, the density of rime layers less than 0.5 mm thick, grown about free-falling frozen drops. These results have been derived from Table 2 of Heymsfield and Pflaum (1985), where they were summarized by averaging out those readings corresponding to several runs performed in similar conditions. They are represented in

Figure 5. As in Fig. 4 but $N$ versus: (a) $K$; (b) $K/\tau$. In (b) dashed and full lines coincide.
Fig. 6 as a function of the parameter X as defined here, i.e. calculated for the droplet impact speed, $v_o$, at the stagnation point of the sphere. In fact, we do not follow the modified expression of X proposed by Pflaum and Pruppacher (1979) and by Heymsfield and Pflaum (1985), who, instead of $v_o$, used the mean droplet impact speed $v_i$, defined as indicated in the introduction. This is because the study of the ice density of fixed cylinders as a function of the angular distance, $\theta$, from the stagnation region (LNP) has shown that the local values of $\rho$ cannot be related to the variations with $\theta$ of the local impact speed $v$. Consequently the value of X has been calculated by evaluating the Stokes number for each group of experiments and deriving $v_o$ from the $v_o(K)$ curve given by Rasmussen and Heymsfield (1985).

The similarity between Pflaum and Pruppacher's results and the present ones is interesting because it points to the importance of the fact that, in both cases, the density measured was that of an initial rime layer with a thickness $d < 1$ mm. This condition is common, apparently, and was more important than the differences between the two experimental systems, especially with respect to the shape and size of the collectors, these in the present case being cylinders of 0.5 cm radius, and in the case of the above-mentioned authors, spherical frozen drops of 100 to 300 $\mu$m radius.

Considering now the experimental points shown in Fig. 6, the $\rho(X)$ behaviour can be analysed in two ranges of variation of the parameter X.

1. We can attempt to find a $\rho(X)$ curve valid over the whole range studied in the present work, i.e. $0.5 < X < 7$, in which case the experimental results can be fitted by curve (c) in Fig. 6, satisfying the equation:

$$\rho = 0.2X^{0.5}. \quad (3)$$

However, it must be noted that, when Eq. (3) is applied, the experimental points are
distributed along the curve with an error which varies between +0.07 and -0.09, except for the point corresponding to \( X = 6.5 \), where the error is +0.17.

This larger error, not justified by the experimental procedure, suggests that the last point, obtained for \( K = 3.2 \), does not belong to the same curve as the other ones, obtained for \( K < 2 \).

2. We can find the best-fit curve only for the experimental points obtained for \( X < 4 \), \( K < 2 \). In this case, it would result that

\[
\rho = 0.2X^{0.4}. \tag{4}
\]

This equation represents satisfactorily all the results in the range considered, and gives, for \( X < 1 \), a slow variation of \( \rho \) with \( X \) that agrees with the apparent tendency of the experimental points to approach a limit value near \( \rho = 0.15 \).

It is interesting to observe that Eq. (4) is quite similar to one that could represent the results of Pflaum and Pruppacher, modified to take into account the present definition of \( X \), namely:

\[
\rho = 0.23X^{0.44}. \tag{4'}
\]

Notice that the use of Eqs. (3), (4) or (4') would not give rise to large differences in the evaluation of \( \rho \) within the interval studied. A larger difference exists, however, between these equations and Eq. (2). Thus, since the measurement methods used in the present work and by Prodi et al. were similar, both for the evaluation of the droplet spectrum and for the densitometric analysis, we can assign this difference in behaviour to the higher values of \( K \) used in the previous work, which could play an additional role in the phenomenon, apart from that related implicitly to the dependence on \( K \) of Macklin's parameter \( X \). This conclusion would also agree with the fact already discussed, that the experimental point for \( X = 6.5, K = 3.2 \) was not represented satisfactorily either by Eq. (4), found in our results for \( K < 2 \), or by Eq. (2) found by Prodi et al. for \( K > 4 \), but was located between the two.

(ii) Local and mean deposit density as a function of rime thickness. A few deposit sections were scanned along circular paths at different radial distances from the section centre, as indicated in section 2. Examples of the diagrams for deposits 042 and 040 are given in Figs. 7 and 8, while Fig. 9 gives similar diagrams, derived from scanning a micrograph taken by Prodi et al. (1986b), for deposit TS20. In the cases of deposits 042 and 040, calibration curves were used to derive \( \rho \) from the densitometer reading, by the method indicated by LNP. For deposit TS20, the micrograph was considered by Prodi et al. (1986b) as remaining inside the linear sensitivity range of the film, and we performed the calculations of \( \rho \) according to this assumption.

Note that in all cases the scans are characterized by oscillations of the signal which consist, for each deposit, of nearly periodic maxima with an approximately constant value and of correlated minima with a depth which increases with the scan radius, i.e. with the thickness \( d \) of the rime layer enclosed by the scan. Since the density variations observed along these scans are evidently related to the deposit morphology, which has been shown to depend on the Stokes number, we will give here, for each example, both the values of \( K \) and of \( X \).

The example in Fig. 7 was obtained for \( K = 0.7, X = 2.7 \). In this case the minima are already pronounced in scan (a), corresponding to \( d = 0.4 \text{mm} \). In scan (b), obtained for \( d = 2 \text{mm} \), the deepest minima reach the baseline of the diagram, indicating that the densitometer light beam intersected zones of the micrograph corresponding to air interstices in the deposit section, i.e. air entrances between lobes. Here, the maxima
Figure 7. Densitometer readings $\Delta I$, or correlated $\rho$, versus angular distance $\theta$ from a given origin, along densitometer scans for different rime thicknesses $d$. Deposit grown with $K = 0.8$, $X = 2.7$: (a) $d = 0.4$ mm; (b) $d = 2$ mm.

corresponded to an ice density ranging from $\rho = 0.3$ for the more internal scan to $\rho = 0.25$ for the more external one.

The example in Fig. 8 was obtained for $K = 3.2$, $X = 6.5$. In this case too, the depth of the minima increases along with $d$ but the phenomenon is less pronounced than in Fig. 7, deep minima only beginning to appear in the second scan, corresponding to $d = 1.4$ mm. In the third scan, corresponding to $d = 2.8$ mm, the minima, though quite deep and more frequent than in Fig. 7(b), do not reach the reference baseline of the diagram, showing that either the air gaps did not cross the whole section or they were too thin to be completely resolved by the scanning light beam. Here the density corresponding to the maxima was 0.72 in the first scan, decreasing to 0.67 in the last scan.

Finally, Fig. 9 shows a micrograph of deposit TS20 and the corresponding densitometer scans. From the data on the growth conditions provided by Prodi et al (1986a) we calculated for this deposit the values of the parameters, $K \approx 4.5$, $X \approx 5$. Note that the depth of the minima in the scans is much less than for similar thicknesses, $d$, of deposit 040, and that even for the diagram corresponding to $d = 6.2$ mm, located at about
1.5 mm from deposit periphery, the few deepest minima are narrow and their bottom remains well above the reference baseline of the scan. Here the density corresponding to the maxima for the three scans was $\rho = 0.75$.

From diagrams of this type we derived for some deposits the mean density, $\bar{\rho}$, corresponding to each scan, as a function of $d$. Typical examples of $\bar{\rho}(d)$ curves are given in Fig. 10. It can be seen that for deposits grown in the course of the present work, corresponding to $K \leq 3$, $\bar{\rho}$ decreases markedly with increasing $d$, becoming, for $d > 1$ mm, more than 30% lower than the density calculated in the zones corresponding to the maxima, i.e. to the lobe interior. However, the decrease of $\bar{\rho}$ is much less pronounced for deposits TS20 and TS18, grown at larger values of $K$, where the difference between the density measured in the maxima and that given in Fig. 10 is $\leq 10\%$.

From the results shown in Fig. 10, we also derived the mean density, $\rho_m$, of each deposit, as a weighted average of $\rho$ calculated by dividing the section into successive rings, each containing a scan, and by assigning to the mean density measured along each scan a weight proportional to the area of the corresponding ring.
Figure 9. Morphology and densitometer scans for deposit TS20 grown by Prodi et al. (1986a) for $K = 5$, $X \approx 5$: (a) X-ray micrograph of a section; (b) and (c) as in Fig. 7 but for $d = 0.8$ mm and $d = 3.6$ mm, respectively.
The results are represented as a function of $X$ in Fig. 11, where Macklin’s curve is also given. Note that the mean density, $\rho_m$, calculated for the deposits grown in the present work, can be considered approximately to satisfy Eq. (1), found by Macklin from mean volume and mass measurements. By contrast, the experimental points obtained for deposits TS20 and TS18, corresponding to $K > 4$, are found to differ significantly from Macklin’s curve. Though only two micrographs of deposits previously grown for $K > 4$ have been analysed here, this difference can be considered systematic, since the micrographs obtained by Prodi et al. for other deposits grown in the same range of $K$ and $X$ show a morphology which is very similar to that of TS18 and TS20.

4. DISCUSSION

(a) Discussion of present results

The results reported in section 3(a) show that the morphology of rotating deposits depends on the Stokes number, which is basically analogous to what was observed previously for fixed deposits. Actually, the formation of well-developed lobes and entrances for $K < 2$, the entrances changing into thin air gaps for $K \approx 3$, the difference between the structures of the deposits grown in the present work for $K \leq 3$ and of the more uniform deposits grown by Prodi et al. (1986a) for $K > 4$, represent a behaviour which can be made to correspond to that already reported for fixed deposits. Actually, in the last case, large lateral voids and concave growth fronts correlated with $K < 2$, while for $K \approx 3$ the lateral voids were reduced to thin air gaps and the growth front became convex in shape. Finally, at $K > 4$ the deposits changed to nearly uniform internal structure, as previously shown by Prodi et al. (1986c).
This analogy is of interest because it permits a qualitative interpretation of the effects observed, without direct comparison with the results of simulation models, which have not been developed at present for rotating collectors. The correlation of the deposit structure with $K$ allows us to establish that the knobby structure of the deposits depends mainly on the shadow effect, which increases with the trajectory curvature of droplets impinging on the collector surface. Thus the formation of entrances between lobes would result from a decrease in the mean collection efficiency on parts of the surface which fell within the shadow of protuberances during a fraction of the rotation cycle. Here the morphology of deposits a few millimetres thick shows that, when stationary growth conditions are attained, the phenomenon becomes approximately periodic, the angular distance between lobes depending on the shadow effect and, therefore, on the value of $K$. This observation agrees with the results in Fig. 5, which show that the lobe angular frequency (proportional to $N$) is a function of $K$ when $\bar{r}$ is constant, and of $K/\bar{r}$ when the droplet spectrum is modified. In the latter case, the fact that the droplet radius had to be introduced explicitly in the reference parameter can be related to the different way droplets of diverse median volume radius pile up and determine both the development of the lobes and the length of their shadow.

Note that the importance of the value of $K$ on deposit morphology, and the correlations existing between morphology and density suggest that the latter cannot be only a function of the parameter $X$, which is not linked to the shadow effect. This fact, already pointed out in section 3(b), will be taken into account in the following discussion of the density behaviour both in the first rime layer and in the whole deposit.

(i) Ice density of the first rime layer and of the lobe interior. It was shown by LNP that the experimental points for the rime density in the stagnation region of fixed deposits were distributed, for $X > 3$, along curve (b) in Fig. 6, derived from Prodi and Levi.
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(1987), and were scattered above this curve for lower values of \( X \). The comparison of these results with those in Fig. 6 indicates that, for the same values of the growth parameters, the stagnation-region density, in the first rime layer and in the lobe interior of rotating cylinders, is higher than that measured here. For instance, low-limit values of \( \rho \) are found in both cases for \( X < 1 \), but these become \( \rho \approx 0.3 \) for fixed deposits and \( \rho \approx 0.15 \) for rotating ones. The analysis of the deposit microstructure indicates that the shadow effect, periodically operating at the feather sides during rotation, is responsible for this behaviour.

Actually, it has been noted in the descriptions of Figs. 1 to 3 that the initial rime layer and the lobe microstructures consist of thin ice feathers and air gaps, the direction of which varies, inside lobes, with the value of \( K \) that determines the droplet impact direction. On the other hand, the behaviour observed by Prodi et al. (1986b), showing that the density of the initial rime layer of their rotating cylinders coincided fairly well with that of the stagnation region of fixed deposits, can be interpreted by assuming that, in their growth conditions, characterized by \( X > 3 \), \( K > 4 \), the shadow effect was negligible. The more pronounced shadow effect, determined by the smaller values of \( K \) used in the present work, was especially evident in the range \( 0.5 < X < 3 \), \( K < 2 \), where the experimental points have been represented by Eq. (4). This effect decreased when the growth parameters increased up to \( X = 6.5 \), \( K = 3.2 \), giving place to the experimental point which, as noted in section 3(b), did not belong to this curve. The above considerations suggest that, if \( X \) and \( K \) were varied independently from each other, different \( \rho(X) \) curves could be obtained for the first rime layer of rotating cylinders, each one corresponding to a given value of \( K \). This could be achieved, for instance, by varying \( T_d \), the other parameters remaining constant. To obtain such curves was not the purpose of the present work, where the growth conditions were initially chosen so as to cover the low-density range not yet reached using similar growth conditions and analysis methods. Some interesting results could possibly be obtained in the future by following this line of studies. However, it must be borne in mind that even the shadow effect is not a simple function of \( K \), because it could not have the same effects when the variables on which \( K \) depends, such as \( R \), \( \tilde{r} \) and \( V \), are varied in different ranges.

For example, we have seen that the results obtained by Pflaum and Pruppacher showed a \( \rho(X) \) behaviour very similar to that observed here. However, the values of \( K \) for their small frozen drops, with radii varying between 100 and 300\( \mu \text{m} \), collecting droplets of 10 or 6\( \mu \text{m} \) median volume radii were substantially larger than for our cylindrical collectors with 0.5 cm radius. In this case, the variation by more than one order of magnitude of the radius \( R \) and, therefore, of the ratio \( \tilde{r}/R \), could have determined a shadow elongation, as shown in Fig. 12, possibly counteracting the opposite effect determined by the increase of \( K \).

It can be concluded that the proposed \( \rho(X) \) equations can only be considered strictly valid in the range of variation of \( K \) and \( \tilde{r}/R \) which was characterized by the previously discussed experimental conditions, and that more complex equations, possibly of the type proposed by Jones (1990), should be applied for a general representation of the phenomenon.

(ii) Mean density of knobby deposits. As shown in Fig. 10, the mean density of rotating deposits can decrease markedly with the deposit thickness, this behaviour being a consequence of the presence of entrances between lobes. Actually, the values of the maxima in the diagrams of Figs. 7 and 8 show that the lobe density, measured in the area centred on the lobe axis, did not change considerably with the rime thickness. Figure 10 shows that, at a few millimetres thickness, the mean density of deposits grown for
$K \leq 3$ is represented by experimental points located near Macklin's curve.

These results suggest that, though the working conditions described by Macklin corresponded to $K$ values varying in a relatively wide range, most of his experiments must have been performed at relatively low values of $K$, possibly included inside a range near that in the present work. Since the external profile of the deposits used by Macklin to calculate the accreted volume might have been similar to the upper deposit profile shown in Fig. 1(a), which evidenced the lobe tips as small roughnesses only, the volume evaluations could have included unseen entrances. Therefore, the mean density obtained could have coincided approximately with that derived here from averaging the density measured along the whole of the scanned areas, including lobes and entrances. In these conditions, Macklin's law (Eq. (1)) can be considered as approximately valid.

However, for larger values of $K$, the curve proposed by Prodi and Levi (1987) would represent the actual behaviour better, because the presence of thin air gaps between lobes modifies the mean density very slightly.

The present considerations could also explain Macklin’s observation that densities higher than the average occurred when his experiments were performed at a temperature of $\leq -30^\circ$C. In fact, when the same value of $X$ is wanted for two experiments performed at markedly different temperatures, the product $rv_o$ must be increased by the same amount as that of the absolute value of $T_d$. Thus, since an increase of $rv_o$ corresponds to an increase of $K$, the experiments performed at the lowest values of $T_d$ had to be conducted under conditions in which the shadow effect was weak and consequently the mean density was high with respect to that found for the same $X$ at higher temperatures.

(b) Discussion of recent results

The results of Jones (1990) show some interesting points in common with those obtained here, so that a discussion of them is worth adding to that of the present results.

Jones’ data were derived from a large series of measurements performed on Mt. Washington for rime deposited on a multicylinder with diameters varying from 0.158 to 7.62 cm. The rime density evaluations were obtained from mass and volume determinations; but, since the rime layers being studied could reach the maximum thickness of 2 mm only on the smallest cylinder, it can be thought that the formation of lobes and entrances had, in these conditions, little effect on the mean density, so that the results can be compared with those obtained here for the first 1 mm thick rime layer.

As an average of the results obtained for the three smallest of these cylinders, the author found for $X < 10$ the equation:

$$\rho = 0.21X^{0.53}$$

(5)
which is not too different from Eq. (3) obtained here for a cylinder of similar diameter. However, the author observed that this was not valid for the three largest cylinders, which gave for \( \rho(X) \) experimental points scattered below the calculated curve. In the light of the present results, this behaviour can be related to the decrease of \( \rho \), occurring at constant \( X \) for decreasing \( K \), as a consequence of the shadow effect. The author, however, does not attempt a phenomenological interpretation of the observed behaviour but proposes a new, more general equation for \( \rho \), represented as a function of several non-dimensional parameters, to which the collection efficiency and the droplet collision speed can be related. These parameters are mainly the Stokes number \( (\pi_K) \); the \( \phi \)-parameter from Langmuir and Blodgett (1946) \( (\pi_\phi) \) and a parameter proportional to the ratio of the convective heat flux and the heat flux due to droplet freezing \( (\pi_C) \). The proposed equation:

\[
\rho = 0.249 - 0.084 \ln \pi_C - 0.00624(\ln \pi_\phi)^2 + 0.135 \ln \pi_K \\
+ 0.0185 \ln \pi_K \ln \pi_\phi - 0.0339(\ln \pi_K)^2 
\]

was found by Jones to give the Mt. Washington results with standard error 0.1 g cm\(^{-3}\).

We have tried to use this equation in conditions typical of those used in the present work. In some cases, the calculations have been performed in supposed conditions, typified by given values of \( X \) and \( K \) (or \( \pi_K \)) and of the geometrical parameters of the system, characterized by the values of \( R \) and \( \bar{r} \), so that \( V \) and \( v_o \) were calculated from the value of \( K \), \( T_d \) from that of \( X \), and values were assigned to \( T_a \) and \( W \) so as to obtain reasonable \( (T_d - T_a) \) differences. The evaluation of \( W \) was only approximate, but, as noted by Jones, an error in this variable does not affect the results very much. Calculations have been performed using \( R = 0.5 \) cm, \( \bar{r} = 10 \) \( \mu \)m. In Table 1, some examples are compared with the present results and with those calculated from Eq. (2).

<table>
<thead>
<tr>
<th>( X )</th>
<th>( K )</th>
<th>( \pi_K )</th>
<th>( \rho ) from Eq. (6)</th>
<th>( \rho ) from present results</th>
<th>( \rho ) from Eq. (2)</th>
</tr>
</thead>
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<tr>
<td>2</td>
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<td>—</td>
<td>—</td>
</tr>
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<td>3</td>
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<td>—</td>
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<td>—</td>
</tr>
<tr>
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<td>4</td>
<td>0.86</td>
<td>—</td>
<td>—</td>
<td>0.86</td>
</tr>
</tbody>
</table>

The table shows that, for constant \( X \), the value of \( \rho \) calculated from Eq. (6) increases with \( K \), as expected according to the present results. However, the calculated values of \( \rho \) are somewhat higher than those derived from the experiments, especially in the range of low densities, as shown in line 1 of the Table. Notice also that the values of \( \rho \) calculated from Eq. (2) would correspond, according to our criterion, to conditions where the shadow effect is negligible, so that \( \rho \) should have its maximum value for the given \( X \); however, for \( X = 2 \), \( K = 3 \), the value of \( \rho \) calculated from Eq. (6) is significantly higher than that from Eq. (2). This suggests that Eq. (6) would need further adjustment.

On the other hand, considering the present results and the previous ones on fixed cylinders (LNP), as well as those obtained by Pflaum and Pruppacher (1979), some limitations in the application of Eq. (6) must be pointed out. Actually, this equation could not be used in the following cases:
1. With a deposit thickness \( d > 1 \text{ mm} \) and conditions favourable for the formation of lobes and protuberances \((K < 3)\). In fact, in this case the mean density along a given scan would decrease with \( d \), as shown in Fig. 10, and the average deposit density could satisfy Eq. (1), as shown in Fig. 11.

2. When the calculated density is that in the stagnation region of fixed deposits, where the shadow effect does not operate, so that the parameter \( K \) only affects the collection efficiency and the droplet impact speed.

3. When the collector is a small particle, such as the frozen drops used by Pfaum and Pruppacher, where the shadow effect for a given \( K \) is possibly enhanced because of the higher curvature of the collector surface, as discussed above.

5. Conclusions

The application of the X-ray micrography technique to the measurement of rime density has permitted us to make a distinction between the ‘local density’, derived for each point of the densitometer reading and the ‘mean density’, which can be obtained by averaging out the densitometer readings along one or more scans of the deposit imprints.

The results have shown that the density of rime grown on rotating collectors reveals a dependence on the growth parameters more complex than had been previously assumed. Actually the density behaviour is determined by two different phenomena, one being the droplet spreading on the substrate, the other being the shadow effect.

It is interesting to observe that Eq. (6), proposed by Jones, implicitly takes into account both these phenomena, though the different parameters introduced in the \( \rho(\pi) \) equation apparently were only thought to affect the collection efficiency (and consequently \( T_d \)) and the impact speed (i.e., \( v_i \)), without taking into consideration the shadow effect that depends directly on \( K \). At any rate, the fact that \( K \) appeared explicitly in the equation, permitted the author to adjust the different coefficients, so that the shadow effect was in some way taken into account.

Considering now the different \( \rho(X) \) and the recent \( \rho(\pi) \) equations, regarding their practical applications, it can be concluded that all of them are somewhat limited to the parameter range where the corresponding study was performed. Thus, considering the first rime layer of rotating cylinders with a diameter more than one order of magnitude larger than that of the impacting droplets, Eq. (2) would be valid for \( K > 4 \), where the shadow effect is negligible, while Eqs. (3) and (4) would be valid for smaller values of \( K \). As for larger deposit thicknesses, \( \rho(X) \) would only differ by less than 20% from Eq. (2), when \( K > 4 \) and large air entrances are not formed between lobes, while for smaller values of \( K \), Eq. (1) could possibly be applied.

On the other hand, for small spherical particles, Eq. (4’) should be used to determine the initial rime density. However, in this case also, it must be noted that this equation does not take into account the density reduction which occurs when lobes and entrances are formed.

Considering now Eq. (6), where \( \rho \) is given as a function of \( \pi \), we have noted, in Table 1, some differences with the present results, especially for the low-density range. A more extended study is needed in order to establish its reliability under different conditions. We expect, however, that it could give satisfactory results if used to calculate the ice density of rime formed on cables or similar structures, but it could not be applied to graupels growing in free fall.
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