Ballistic accretion on seeds of different sizes

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Computer simulations of ballistic driven accretions on seeds of different sizes were carried out. The results, in agreement with previous work reported for the general case of seeds of particle size, show that the kinematics of accretion does not depend on the size of the seed. Specific morphological properties (growth angle and channel orientation) are investigated and found to depend only on the elementary aggregation process.

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I. INTRODUCTION

The increasing interest in the study of ballistic accretion of materials is due to the combination of growing computer capabilities, theoretical curiosity, and a wide range of possible applications. Although various models have been developed, like diffusion-limited aggregation (DLA) and ballistic driven aggregation (BDA), they are difficult to handle analytically and most of the work must be based on computer simulation. The widening use of more powerful and dedicated computers (array processors) has expanded the possibilities of numerical simulations.

Theoretical curiosity is being stimulated by the interesting results being attained in scaling behavior and fractal effects of deposits. The range of potential applications is wide, including atmospheric ice accretion (as in the present study), thin film growth, colloidal aggregation, metal growth from melted droplets, properties of columnar and feathery structures, and almost any structure formed by addition of subunits from the outside.

A deterministic treatment of ballistic aggregation on a point seed, based on the assumption of the tangent rule determining the mean direction of growth, is reported by Ramanlal and Sander [1]. This rule relates the angle $\alpha$ between the incident particle direction and the normal to the accretion front and the column angle $\beta$. From this assumption they determine empirically by computer simulation the limiting fan angle ($\theta_c = \alpha - \beta = 19.5^\circ$) and density as a function of the angle.

Analytical calculations performed by Limaye and Amritkar [2] report the probability distribution for width and height of the resulting fanlike structure. The width is derived exactly while height is found to depend on the nature of the top surface: a fan angle of about $20^\circ$ and limiting density value of 0.5 are confirmed. The problem is handled in terms of transition probability: the width increases from $w'$ to $w$ in the time interval $\tau$, the model being based on probability distribution as a Markov process.

Rambaldi, Prodi, and Porcu [3] also report a probabilistic theory of ballistic aggregation on a seed based on the probability function of the elementary accretion process: the theoretical angle of $19.15^\circ$ agrees with the numerical simulation for circular particles.

The present paper reports numerical simulations of atmospheric icing in which supercooled droplets impinge on tiny obstacles. Accretion is simulated on a plane where circular particles (of radius $r$), randomly starting from infinity, move continuously along rectilinear trajectories. The radius $R$ of the circular seed of accretion is a multiple of the radius of accreting particles; the seeds in our experiments are 5, 10, 20, 30, 50, and 80 times the particle radius.

Two types of experiments were carried out: growth of individual aggregates of about 50 000 particles and cumulation of 1000 aggregates of 10 000 particles per each $R$.

II. NUMERICAL RESULTS

The results can be discussed in terms of morphology (fan angle, shape of the profile), surface density and channel orientation, for individual and cumulated aggregates.

A. Individual aggregates

The sequence of results given in Fig. 1 is in order of progressive seed to particle ratio. Two density measures are computed for an individual aggregate on a seed with $R = 10r$: the first [Fig. 2(a)] is the two-dimensional density sampled on circular corona sectors (whose angular width is determined for each sector by the position of the leftmost and rightmost collected particles) by increasing the distance from the seed; the second [Fig. 2(b)] is the angular distribution of the density ($\Delta \theta = 1^\circ$ is the sector width). The density curves show a great variability: the corona density for an individual aggregate varies in correspondence to large channels, broken branches, and local shadowing effects. A better understanding of the behavior of the single aggregate is possible by means of
the angular density (Liang and Kadanoff [4]): the density inside is almost constant but as the aggregate's edge is approached deep throats of density occur, corresponding to channels. Moreover, the sudden drop in density at about $\theta = 20^\circ$ defines the fan limiting angle. Another measure of the fan angle is given in Fig. 2(c): for each corona considered for the sampling in Fig. 2(a), the left and right angles in respect to the direction of particle trajectory are plotted. The positions of the external particles give values of the local fan width's rapidly varying behavior, which depends on the inherent fuzziness of the single aggregate border.

Because of the role played by the channels in the morphology and density of an aggregate, a semiautomated procedure for channel evaluation is adopted as follows: (i) the continuous particle distribution is sampled on a grid $r \times R$ ($r=1$ is the radius of incident particles); (ii) a subjective identification of channels is performed (we chose closed channels); (iii) the selected channel is extracted, its boundaries are defined, and the characteristic features of the channel are computed e.g., center-of-mass position in the fan, area, ellipticity, and angle between the major axis and the direction of the particles.

Figure 3 shows, as a preliminary result, the distribution of the channel angles, which confirms the leading role of the fan angle $\theta = 19^\circ$ in the kinematics of accretion.

### B. Cumulation of aggregates

In order to smoothen out the variability of the single aggregate behavior in calculating general properties of the growth, we considered cumulations of 1000 aggregates at the different seed to particle ratios. In Fig. 4 the average angular density for the case with $R=r$ is plotted together with its first derivative absolute value: as pointed out above, we chose as the limiting fan angle the value for $\theta$ in which the slope of the density function is maximum. We found $\theta = 19^\circ$ for each seed to particle ratio. The corresponding threshold for the limiting density is 0.2, and its maximum value 0.36, in the direction of particle motion ($R=r$). The radial density profile performed on individual accretion shows that there is no plateau of density in the inner deposit, but rather a gradual rise to the maximum value. This is due to the lower, but not zero, probability of having large channels in the inner part of a deposit.

Figure 5 shows the results of all the experiments for the same seed to particle ratios as in Fig. 1. The shades of gray in the images are related to the local value and slope of the density: the lightest gray represents cells with low particle occurrence, medium gray cells in which the local density is rapidly varying, and the dark gray cells with high density and low local variability. The fact that the opening angle of a fan does not depend on the size of the seed can be seen in Fig. 6: we overlapped the six cumulated deposits of Fig. 5 (thresholded with the limit density for $\theta = 19^\circ$) by scaling forward the relative positions of the seeds with a factor $R/tan(\theta)$.

### III. Discussion and conclusion

We have investigated the properties of ballistic accretion on multiple seed to particle ratio on a plane. The main conclusion from the examination of the deposit behavior is that the features are determined at the elementary level of accretion, as pointed out by Rambaldi, Prodi, and Porcu: our experiments indicate that when sufficiently far from the seed, the memory of the seed is lost, and the opening fan angle and the density depend on the geometry of the single aggregation process. Application of these results is in time growth of supercooled droplets that freeze on impact at low temperatures on obstacles of various sizes. One purpose of numerical simulation is to compare resulting data to tunnel accretion experiments, since we are capable of generating monodisperse water droplets in high concentration (Porcu and Prodi [5]).

A natural extension of this application will be to introduce curved trajectories for the droplets, to account for the hydrodynamics of the flow around the obstacle, and to introduce a more realistic third dimension on both a spherical obstacle and a wire (infinite cylinders) as seeds.

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